Answers to written exam at the Department of Economics winter 2019-20

Economics of the Environment, Natural Resources and Climate Change

Final exam

9 January 2020

(3-hour closed book exam)

Answers only in English.

This exam question consists of 4 pages in total

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Exercise 1. Climate Policy and The Green Paradox

The model examined here is basically the same as the model investigated in Lecture Note 16. The only difference is that the carbon tax is placed on the consumers of fossil fuels instead of the producers.

In the following dots over variables denote derivatives with respect to time.

Answer to Question 1.1

The current value Hamiltonian is given by:

$$H = P_t R_t - \lambda_t R_t.$$

The first-order conditions - including the transversality condition (TVC) - are:

$$\frac{\partial H}{\partial R_t} = P_t - \lambda_t = 0 \quad \Leftrightarrow \quad P_t = \lambda_t. \tag{i}$$

$$-\frac{\partial H}{\partial S_t} = \dot{\lambda}_t - \lambda_t r = 0 \quad \Leftrightarrow \quad \frac{\dot{\lambda}_t}{\lambda_t} = r. \tag{ii}$$

$$\lim_{t \to \infty} \lambda_t S_t \mathrm{e}^{-rt} = 0. \tag{TVC}$$

The TVC states that, in the very long run, the present value of the remaining fossil fuel reserve must be zero. This is clear since λ_t is the value of one unit of fossil fuel in the ground at time t for the representative mining firm. Meanwhile, $\lambda_t \mathrm{e}^{-rt}$ is the present value of that unit of fossil fuel. If the present value of fossil fuels remain strictly positive over the investigated period, the TVC implies that the entire fossil fuel stock will be extracted in the very long run.

Answer to Question 1.2

From (i) it follows that:

$$P_t = \lambda_t \quad \Rightarrow \quad \dot{P}_t = \dot{\lambda}_t. \tag{iii}$$

From (i), (ii) and (iii) it follows directly that:

$$\frac{\dot{P}_t}{P_t} = r. \tag{Hotelling rule}$$

The Hotelling rule reflects the optimal portfolio choice of the mining firm. This is easier to see when the rule is formulated as:

$$\dot{P}_t = P_t r.$$

On the margin, the mining firm has two options. The first option is to extract and sell one additional unit of the natural resource, invest the revenue, P_t, and obtain the return r per unit of revenue. This amounts the the right-hand side of the above equation. The other option is to let the same unit of the resource stay in the ground. This will give the firm the return $\dot{P}_t$.

If $\dot{P}_t < P_t r$ the mining firm prefers the first option, implying that the mining firm will extract as much as possible. If $\dot{P}_t > P_t r$ the mining firm does not extract any resource. A bounded and strictly positive extraction intensity requires that: $\dot{P}_t = P_t r$. In this case, the mining firm is indifferent between extracting an extra unit of the resource and leaving it in the ground.

Answer to Question 1.3

Firstly, note that:

$$P_t \theta_t = P_t \left(1 + \tau_t / P_t\right) = P_t + \tau_t.$$

Insert $P_t \theta_t$ into the expression for R_t^d:

$$R_t^d = (P_t \theta_t)^{-\gamma}.$$

From the Hotelling rule it follows that:

$$\dot{P}_t = P_t r \quad \Rightarrow \quad P_t = P_0 e^{rt}.$$

Insert this expression together with the expression for θ_t into the above expression for R_t^d:

$$R_t^d = (P_0 \theta_0)^{-\gamma} \mathrm{e}^{-\gamma(r+g)t}.$$

In the next step use that:

$$S_0 = \int_0^\infty R_t \,\mathrm{d}t. \tag{iv}$$

The physical laws ensure that S_0 is greater than or equal to the right-hand side of (iv). Meanwhile, the TVC and the Hotelling rule together imply that in equilibrium:

$$\lim_{t\to\infty} \lambda_t S_t \mathrm{e}^{-rt} = \lim_{t\to\infty} P_t S_t \mathrm{e}^{-rt} = \lim_{t\to\infty} P_0 S_t = P_0 \lim_{t\to\infty} S_t = 0,$$

where it is used that P_t is strictly positive for all strictly positive extraction intensities due to the demand function. The last equality implies that all of the resource must be extracted in the very long run. Accordingly, S_0 must equal the right-hand side of (iv).

Next, it must be such that supply and demand equals in equilibrium: $R_t = R_t^d$. Thus, we can insert the expression for R_t^d into (iv):

$$\begin{aligned} S_0 &= \int_0^\infty (P_0\theta_0)^{-\gamma} \mathrm{e}^{-\gamma(r+g)t} \,\mathrm{d}t \\ &= (P_0\theta_0)^{-\gamma} \int_0^\infty \mathrm{e}^{-\gamma(r+g)t} \,\mathrm{d}t \\ &= \frac{(P_0\theta_0)^{-\gamma}}{\gamma(r+g)}. \end{aligned}$$

This expression is reformulated as:

$$P_0^{-\gamma} = S_0 \theta_0^\gamma \gamma(r+g).$$

Inserting this expression into the expression for R_t^d - which equals R_t - yields:

$$R_t = \underbrace{S_0\theta_0^\gamma\gamma(r+g)}_{P_0^{-\gamma}} \theta_0^{-\gamma} \mathrm{e}^{-\gamma(r+g)t} = S_0\gamma(r+g)\mathrm{e}^{-\gamma(r+g)t}.$$

Answer to Question 1.4

The impact on short-run emissions can be found by taking the derivative of R_0 with respect to g:

$$\frac{\partial R_0}{\partial g} = \frac{\partial}{\partial g}\left(S_0\gamma(r+g)\right) = S_0\gamma > 0.$$

The above expression shows that initial emissions are increasing in g. Thus a faster growth rate of the tax wedge - implying a faster growing carbon tax - results in higher short-run emissions.

Intuitively, a higher growth rate of the tax wedge implies that future fossil fuel demand is dampened relative to current demand. This is because the initial wedge is unchanged, and thus, the higher growth rate increases the consumer price of fossil fuels in the future relative to the present.

As a consequence, fossil fuel producers receive a lower price per unit of fossil fuel in the future for a given extraction path. Thus they have an incentive to extract more closer to the present - where demand is relatively higher - when the growth rate of the wedge is increased.

As the supply of fossil fuels increases in the short-run so does the consumption of fossil fuels and thereby carbon emissions.

Answer to Question 1.5

Insert the expression for x_t into the expression for D_0:

$$D_0 = \int_0^\infty x_0 R_t^d \mathrm{e}^{(\delta-\rho)t}\,\mathrm{d}t.$$

Insert the expression for R_t - which equals R_t^d - derived in Question 1.3 (the expression is also stated directly in the assignment text):

$$\begin{aligned} D_0 &= \int_0^\infty x_0 \underbrace{S_0\gamma(r+g)\mathrm{e}^{-\gamma(r+g)t}}_{R_t=R_t^d}\mathrm{e}^{-(\rho-\delta)t}\,\mathrm{d}t \\ &= x_0 S_0\gamma(r+g)\int_0^\infty \mathrm{e}^{(\delta-\rho-\gamma(r+g))t}\,\mathrm{d}t \\ &= (r+g)\frac{x_0 S_0 \gamma}{\rho-\delta+\gamma\left(r+g\right)}. \end{aligned}$$

Answer to Question 1.6

Differentiating D_0 with respect to g:

$$\frac{\partial D_0}{\partial g} = \gamma x_0 S_0 \frac{\rho-\delta}{\left(\rho-\delta+\gamma\left(r+g\right)\right)^2} > 0.$$

Note that the expression is positive, as we assumed that $\rho > \delta$.

As damage costs increase at a rate slower than the social rate of time preferences, society will prefer to postpone emissions.

The policy considered here does not change long-run accumulated emissions which is given by S_0. But the policy pushes emissions more towards the present, cf. Question 1.4. Thus, the policy increases the present value of the climate damage cost.

Answer to Question 1.7

Denote initial emissions resulting from Policy 1 by R_0^1 and initial emissions resulting from Policy 2 by R_0^2. It follows from the expression for R_t derived in Question 1.3 (and stated in the assignment text) that:

$$R_0^1 = S_0\gamma r$$
$$R_0^2 = S_0\gamma(r+\phi).$$

Here it is used that g can be substituted directly with either 0 or ϕ in the formula for R_t derived in Question 1.3.

Policy 2 is preferred if initial emissions are lower compared to the initial emissions of Policy 1. This is true if the following expression is true:

$$R_0^1 > R_0^2.$$

Inserting the above equations into the inequality yields:

$$S_0\gamma r > S_0\gamma(r+\phi) \quad \Leftrightarrow \quad \phi < 0.$$

This inequality is true by assumption. Thus Policy 2 is preferred.

Intuitively, if $\phi < 0$ then the tax wedge is decreasing over time. This means that the present cost at time $t = 0$ of purchasing one unit of fossil fuel at time t, $\tilde{P}_t \equiv P_t\bar{\theta}\mathrm{e}^{\phi t}\mathrm{e}^{-rt}$, is decreasing over time.[1] To see this:

$$\tilde{P}_t \equiv P_t\bar{\theta}\mathrm{e}^{-(r-\phi)t} = P_0\bar{\theta}\mathrm{e}^{\phi t} \quad \Rightarrow \quad \frac{\dot{\tilde{P}}_t}{\tilde{P}_t} = \phi < 0.$$

Thus, compared to Policy 1, where the tax wedge is constant over time, Policy 2 increases the incentive to postpone extraction, resulting in lower short-run emissions. As the government

[1]The current cost is $P_t\theta_t$. This cost is discounted by e^{-rt} to obtain the present cost.

wants to reduce short-run emissions, it should choose Policy 2.

One (unimportant) detail that is easily overlooked is that Policy 2 eventually results in a fossil fuel subsidy. As stated above $\theta_t = 1 + \tau_t/P_t$, implying that θ_t can only become smaller than one if τ_t is negative, which corresponds to a subsidy to carbon emissions. As $\theta_0 = \bar{\theta} > 1$ the initial per unit carbon tax is positive, and since Policy 2 implies that $\theta_t \to 0$ for $t \to \infty$, the per unit carbon tax will eventually be negative.

Answer to Question 1.8

Some typical examples are:

1. Policies that reduce the effective fossil fuel reserve: leaving more reserves in the ground forever.

2. A global cap-and-trade system.

3. Carbon capture and storage.

4. Afforestation.

5. A per unit carbon tax that grows slower than the fossil fuel price.

The first policy could involve subsidies to fossil fuel producers to reserves left in the ground. Another possibility is to forbid extraction of certain oil or gas fields. The idea is to reduce the stock of fossil fuels that is eventually extracted. Reductions to the effective fossil fuel stock will reduce extraction at all points in time.

The second option ensures a cap over global emissions which also puts a cap over fossil fuels demanded. This will force producers to leave fossil fuel reserves in the ground, reducing the effective fossil fuel reserve.

The third and fourth options take carbon out of the atmosphere. Such policies will reduce the carbon concentration in the atmosphere without distorting the economic incentives of fossil fuel producers.

The fifth option is to introduce a slowly growing per unit carbon tax. If the per unit carbon tax grows slower than the fossil fuel price, the implied ad valorem tax (and tax wedge) is decreasing over time. Accordingly, fossil fuel producers have an incentive to push extractions into the future. This will reduce short-run emissions and decelerate climate change.

Exercise 2: Genuine Savings and Sustainability

Answer to Question 2.1

Genuine saving measures the total increase in the value of society's asset stock. This includes saving in physical capital, human capital, and natural capital. The last quantity can include the change in the value of exhaustible and renewable natural resources. An example of the former is a change in the value of remaining oil reserves, whist an example of the latter is a change in the value of the stock of fish in the oceans.

Genuine saving is used to measure sustainability as discussed below. Essentially, a positive genuine saving implies that the total wealth of society is increasing and vice versa.

Answer to Question 2.2

There are various ways to define a sustainable development. The curriculum deals with three different definitions: (1) non-declining instantaneous utility, (2) non-declining present value of utility, and (3) non-declining wealth. It can be shown that fulfilment of (2) ensures fulfilment of (1), and that fulfilment of (3) ensures fulfilment of both (1) and (2) assuming that future generations use the total stock of wealth optimally.

If genuine saving is zero, total wealth is constant which ensures that (3) is fulfilled. Thus, society can obtain a sustainable development if it ensures that genuine saving is positive.

This result is intimately linked to the Hartwick rule. The rule demands that all rent from natural resource extraction must be invested in man-made capital. In that way, the declining value of the natural resource stock is offset by an increasing value of the capital stock. The two effects balance out such that genuine saving is unaffected.

Other potential discussion points include the substitutability of man-made and natural capital, negative genuine saving as an early warning indicator for an unsustainable development, and the relationship between genuine saving and the Green National Product.